From Rinderle B to the Chipstead High Point Scoring method, version 3

Malcolm Clark

November 20, 2007

Contents

Abstract

The Chips scoring method, which is now implemented within Sailwave (Jenkins, 2006), is derived from the 'Rinderle B' lookup tables (still used by some organisations). This paper examines the evolution of the CHIPS scoring method to its current version, attempting to explain some of the rationale behind the changes. Although the underlying mathematics is examined in some detail, it is discussed and illustrated at a level which should be approachable by even the most number shy. The system offers many advantages for longer series, where the participants need not complete every race to qualify, which tends to be the case for much club yacht and dinghy racing.

1 Introduction

In an earlier paper (Clark, 2006) I discussed some features of the Chipstead High Point Scoring system (CHIPS), developed by Burrell (2006). Attention to some minor anomalies in the that system has led to modification of the parameters although the fundamental structure of the CHIPS equation remains unchanged. This paper traces the origin of CHIPS from its precursor, the Rinderle lookup table of the 80s. This lookup table was originally parameterised by Geoff Burrell. We will examine the features of the Rinderle lookup table and address some of the ways in which it differs from its parameterised form (the 'modified Rinderle B'). We examine the development of the CHIPS scoring systems and explore some of the consequences of the structure of the equation, attempting to show how straightforward the equation is and how the actual scores may be found easily. Lastly,the superficially similar Cox-Sprague scoring system is examined. Results from the use of these various scoring systems, and the prevalant ISAF system are presented and compared.

1.1 Nomenclature

Scoring in sailing (see, for example RYA (2004)) recognises a number of categories to accommodate those who are not explicitly placed in the race: for example, DNF (for Did Not Finish), DNC (for Did Not Come, i. e. to the race), DSQ (for DiSQualified), RAF (Retired After Finishing) and so on.

Bemis (1960) classifies scoring systems into four types, distinguished by the nature of their 'spread'. Spread is defined as the "difference in points assigned to adjacent finishing positions". This paper uses the term 'increment' and the symbol δ for the concept. The four types are:

i) Constant spread systems: where all spreads are the same in all races. Where bonus points are given for the first few places, Bemis regards this as a variation on a constant spread, especially where the value of the bonus is less than the 'standard' spread value.
ii) Increasing spread systems: the spread increases as first place is approached, but is the same in all races (in theory the spread could also decrease, but this was not observed in practise by Bemis).

iii) Variable spread systems: the spread is constant within a race, but varies between races.
iv) Variable *and* increasing spread systems: combining increasing spread and variable spread, the spread between adjacent scores in a given race varies, *and* the spread is different between races.

Bemis also uses two other useful terms to describe the series:

i) a **set** series is one in which a yacht is expected to sail in all races and there is no compensation in the score if she fails to do so; also described as a **compulsory** series by Bavier (1956); and
ii) a **selective** series where a number or proportion of races may be omitted without penalty – a **voluntary** series in Bavier's terminology.

The scoring systems which are considered here are all approached from the assumption that we are scoring a selective (or voluntary) series.

The successive versions of the Chips formulæ are termed Chips1, Chips2 and Chips3. The current formulation is Chips3.

A predecessor of Chips is the Rinderle B lookup table. The formula of the parameterised Rinderle B lookup table is referred to as the 'modified Rinderle B'.

The notation $\mathcal{S}_{p,n}$ refers to the score for *finishing* in position p in a race for which there are n starters.

2 Rinderle B

2.1 The Rinderle B lookup table

The Chips scoring schemes owe their origin to a system known as 'Rinderle B'. Its origins are hazy. Since it is not mentioned in Bemis's (1960) excellent and comprehensive 'Yacht Race Scoring' it is almost certainly more recent than that book. It is (or has been) used by a number of yacht clubs, especially in the USA, where their results refer to the scoring system, but none I have come across provide a source, except in a very general way.[1] The first 'published' reference I have uncovered is Downing (2005) where it described as 'often represented by a lookup table'. An unpublished document (NEMA, 2000?) states that the scoring system was devised in the early 80s by 'Jim Rinderle of Marblehead', who 'developed a series of candidate scoring tables'. In it the goals of the system are said to be to:

i) preserve a bonus for winning;

[1]Internet search engines have provided whatever prior evidence I have. They are seemingly excellent at finding the specific terms, but the majority of suggestions are either very mundane, or are tantalising leads which turn out to be broken links. One general conclusion we can draw is that the system predates the Internet, and any material describing it just never made it into electronic form. It also bears out Bemis's (1960) description of scoring as the '"forgotten man" of yacht racing' that it is absent from Wikipedia, otherwise the repository of much obscure ephemera.

Table 1: Rinderle B scoring table, GMORA (undated), corrected.

place, p \ starters, n	1	2	3	4	5	6	7	8	9	10	11	12	13
1	79.1	81.3	83.6	85.8	87.8	89.6	91.2	92.6	93.9	95	96	96.9	97.6
2		10.5	43	56.8	65	70.7	75	78.5	81.3	83.6	85.6	87.3	88.7
3			10.5	33.7	46.9	55.7	62.1	67.1	71.2	74.5	77.3	79.6	81.6
4				10.5	28.7	40.6	49.2	55.8	61.1	65.3	68.9	71.9	74.5
5					10.5	25.6	36.3	44.5	50.9	56.2	60.6	64.3	67.4
6						10.5	23.4	33.2	40.8	47.1	52.2	56.6	60.3
7							10.5	21.8	30.7	37.9	43.9	48.9	53.2
8								10.5	20.6	28.8	35.5	41.2	46.1
9									10.5	19.6	27.2	33.5	38.9
10										10.5	18.8	25.9	31.8
11											10.5	18.2	24.7
12												10.5	17.6
13													10.5

place, p \ starters, n	14	15	16	17	18	19	20	21	22	23	24	25
1	98.2	98.7	99.2	99.5	99.8	100	100.1	100.4	100.5	100.5		
2	89.9	91	91.9	92.7	93.3	93.9	94.3	94.7	95.1	95.4	95.6	95.8
3	83.3	84.8	86.1	87.2	88.1	89	89.7	90.3	90.8	91.3	91.7	92.1
4	76.7	78.6	80.3	81.7	83	84.1	85	85.9	86.6	87.3	87.9	88.4
5	70.1	72.4	74.5	76.2	77.8	79.1	80.4	81.4	82.4	83.2	84	84.7
6	63.5	66.2	68.6	70.7	72.6	74.2	75.7	77	78.2	79.2	80.1	81
7	56.8	60	62.8	65.3	67.4	69.3	71	72.6	73.9	75.2	76.3	77.3
8	50.2	53.8	57	59.8	62.3	64.4	66.4	68.1	69.7	71.1	72.4	73.6
9	43.6	47.7	51.2	54.3	57.1	59.5	61.7	63.7	65.5	67.1	68.5	69.9
10	37	41.5	45.4	48.8	51.9	54.6	57.1	59.3	61.2	63	64.7	66.2
11	30.4	35.3	39.6	43.4	46.7	49.7	52.4	54.8	57	59	60.8	62.4
12	23.7	29.1	33.8	37.9	41.6	44.8	47.8	50.4	52.8	55	56.9	58.7
13	17.1	22.9	27.9	32.4	36.4	39.9	43.1	46	48.6	50.9	53.1	55
14	10.5	16.7	22.1	26.9	31.2	35	38.4	41.5	44.3	46.9	49.2	51.3
15		10.5	16.3	21.5	26	30.1	33.8	37.1	40.1	42.8	45.3	47.6
16			10.5	16	20.9	25.2	29.1	32.7	35.9	38.8	41.5	43.9
17				10.5	15.7	20.3	24.5	28.2	31.6	34.7	37.6	40.2
18					10.5	15.4	19.8	23.8	27.4	30.7	33.7	36.5
19						10.5	15.2	19.4	23.2	26.7	29.8	32.8
20							10.5	14.9	19	22.6	26	29.1
21								10.5	14.7	18.6	22.1	25.3
22									10.5	14.5	18.2	21.6
23										10.5	14.4	17.9
24											10.5	14.2
25												10.5

ii) have a scoring system that reflects the fact that it is harder to be first among 8 than to be first among 4;

iii) have a scoring system that reflects the fact that it is more difficult for an average racer to achieve a mid-fleet finish in a small fleet because small fleets usually include a large fraction of the best sailors (e.g. bad weather results in a small fleet because the weaker sailors stay at home);

iv) create a scoring system that creates an incentive to race.

The inference from the reference to 'tables' is that the original scheme was not presented as a formula but as a table.[2] Tables are given in Downing (2005),[3] and a fuller version in GMORA (undated).[4] To all intents and purposes these are identical.[5] This latter table is reproduced as Table 1,

[2] In fact, Burrell (2007a) states that "The Rinderle B scores are available only in the form of a table" and references GMORA (undated), the source of which is unfortunately no longer available.

[3] Up to a fleet size of 18.

[4] Up to a fleet size of 25.

[5] There are a small number of disagreements, the most notable at $\mathcal{S}_{2,10}$, where GMORA (undated) has a value of 83.0 and Downing (2005) has 83.6. The latter is more likely to be correct, the mistake a transcription error of a zero for a six. Other discrepancies can

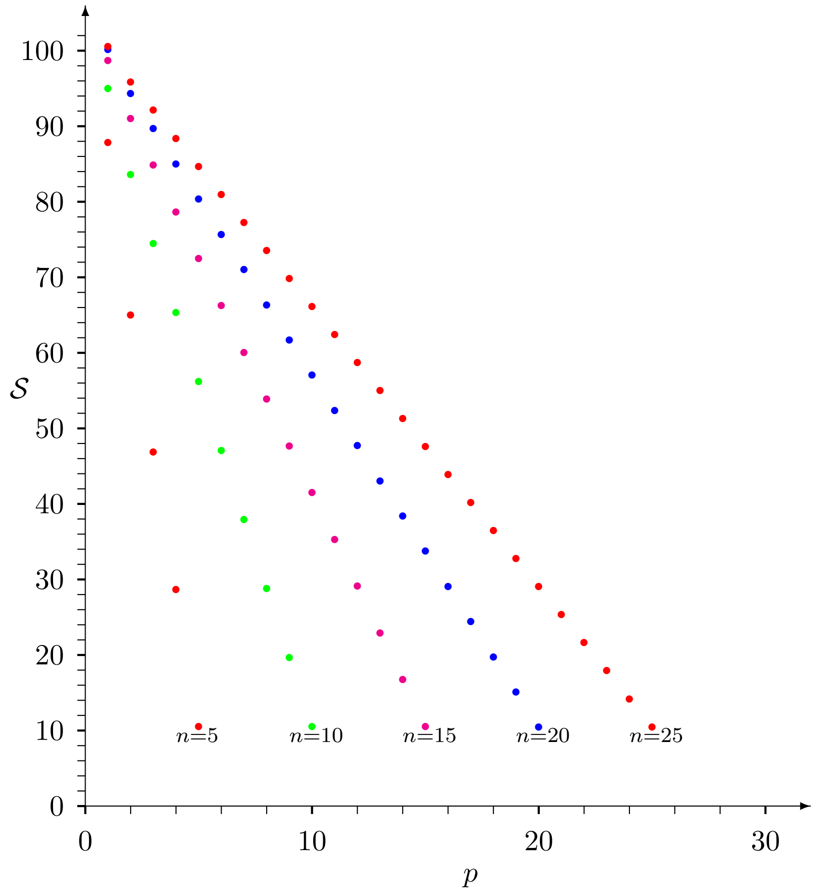

Figure 1: Scores $\mathcal{S}$ for place p in selected fleet sizes n, from the Rinderle B lookup table in GMORA (undated).

but with a correction at $\mathcal{S}_{2,10}$. Note that all non-integer values in his table are given to a single place of decimals – it is probably fair to assume that all the values in the table are rounded to the nearest 0.1 and that the apparent integer values are actually rounded.[6] The score for last place is 10.5;[7] and the highest score for a first place is at least 100.5.

Inspection shows that this is an example of a Bemis 'variable scoring system' since the spread is constant within a race but varies between races. The interval between any two scores $\mathcal{S}_{p,n}$ and $\mathcal{S}_{p+1,n}$ is constant *except* in respect of $\mathcal{S}_{1,n}$ and $\mathcal{S}_{2,n}$, where it is around 1.25 times greater. This is presumably the way that the scoring system 'preserve(d) a bonus for winning'.

be ascribed to rounding. Excluding the gross discrepancy of 0.6, out of the 10 differences in the scores up to a fleet size of 18, all of ±0.1, 6 are positive and 4 are negative. The differences in the two tables point to the existence of an earlier table from which these were (ultimately) derived.

[6] The values in the Downing (2005) table are presented to one decimal place, but were clearly stored in a spreadsheet where the same format was applied to al the cells.

[7] DSC is apparently scored as zero, but the other categories are not explicitly defined.

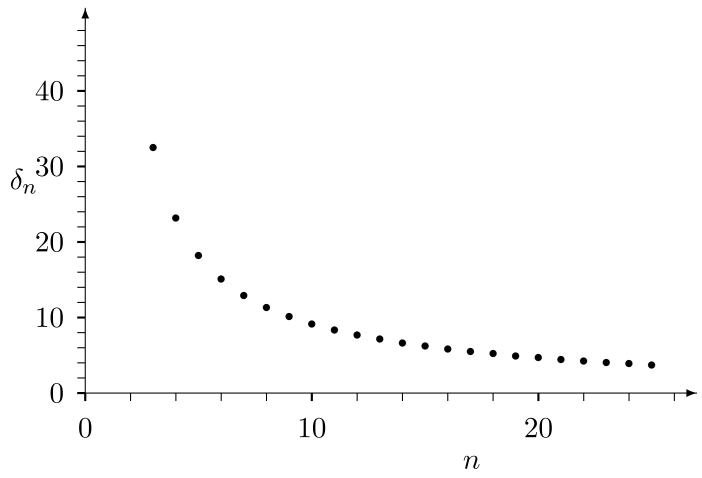

Figure 2: Score interval δ_n derived from the Rinderle B lookup table in GMORA (undated).

Figure 1 shows the scores for selected fleet sizes.

Defining δ_n for $p > 1$ as:

$$\delta_n = \mathcal{S}_{p,n} - \mathcal{S}_{p+1,n} \tag{1}$$

for each fleet size n we expect δ_n to be constant. We could average the δ_n values, but since they are (apparently) rounded to 1 decimal place this is not ideal. We estimate δ_n a little more accurately by calculating the interval as

$$\delta_n = \frac{\mathcal{S}_{2,n} - 10.5}{n - 2} \tag{2}$$

This is of course sensitive to any discrepancies in $\mathcal{S}_{2,n}$. The values of δ_n derived in this way are given in Figure 2.

If we then calculate

$$\delta'_n = \frac{\mathcal{S}_{1,n} - \mathcal{S}_{2,n}}{\delta_n} \tag{3}$$

we obtain a set of values for δ'_n in the range 1.24–1.27 which bears out the premise that first place is weighted by 1.25. This is the evidence which encourages belief in the correction to $\mathcal{S}_{2,10}$ in the GMORA (undated) table.[8]

The main points here are that

- there is an underlying regularity to the Rinderle B look up table:
- the increments for a given fleet size are constant (allowing for the weighting given the first score);
- the intervals δ_n between scores for successive fleet sizes follow a broadly inverse form, as expected from Equation 2.

[8]However, note that as of 2006, the Gulf of Maine Ocean Racing Association were still using the 'uncorrected' value in their scoring (see their race results at `www.gmora.org`).[any UK sites?]

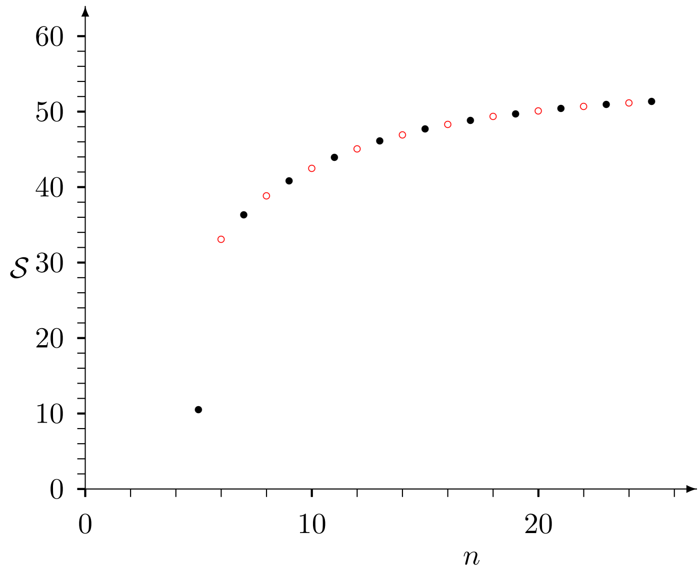

Figure 3: Score for 'average' racer as fleet size n increases. The • are exact values, but the ∘ are interpolated. The first four places are taken by the 'best sailors'; the 'average' racer is placed in the middle of the remaining places.

2.1.1 Does Rinderle B achieve its aims?

Earlier, we noted the goals of the scoring system.

i) It is clear that the first goal, preserving the bonus for winning, is attained by the 1.25 interval 'multiplier' (or weighting) awarded only to a first place;

ii) the second goal is achieved by the way that the score for a first is dependent on fleet size (see Figure 4); the larger the fleet the higher the score (apparently);

iii) the third goal appears to seek to reward 'the average racer' who may be squeezed out of mid-fleet points because small fleets have a higher proportion of 'the best sailors' who are prepared to sail in all weathers, but it is difficult to see just how to demonstrate this. Imagine the scenario where the first four places are regularly taken by our best sailors. Our average racer is placed in the middle of the remaining places. As fleets increase in size his score is modeled in Figure 3. It is not obvious that he is rewarded.

iv) the 'incentive to race' goal is not explicitly achieved. Awarding a minimum score of 10.5 to a finisher is not an automatic incentive to get him or her out on the water, especially if we do not know what the alternatives are. Burrell (2007a) states that the score for DNF is 5, and we assume (on the basis of published results) that the score for not racing is simply zero. In small fleet sizes, coming last, or (worse?) not

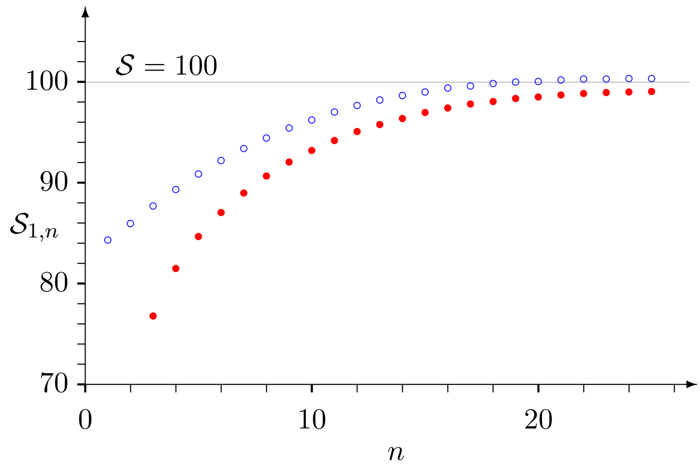

Figure 4: Score for first ∘ and 'adjusted' first • for fleet size n. The adjusted scores are given from $\mathcal{S}_{1,3}$ only.

finishing is rewarded only slightly better than not appearing at all. On the other hand, in small fleet sizes, coming second last does get some useful points. For example, $\mathcal{S}_{3,4}$ scores the same as $\mathcal{S}_{18,24}$ (the value is actually 33.7).[9]

2.1.2 Maximum scores

What is the maximum score attainable with the Rinderle B lookup table? The highest published value (from the table) is 100.5, but this is for a fleet size of 25. Would larger fleet sizes give successively larger scores for first place? In the absence of a clear and explicit structure for the relationship between scores, we can have recourse only to speculation. However, this does illustrate a potential limitation of the lookup table: it is restricted to fleet sizes no greater than 25. In reality, this is unlikely to be an issue.[10] The table was apparently designed for ocean racing where a number of races took place over (say) a year. While more than 25 yachts might take part in the whole series, it is unlikely that any one race would attract this number. For example in 2006 the Gulf of Maine Ocean Racing Association had 38 yachts in its largest class, Class A, but the maximum number who raced against one another in any one race was less than half of this.

[9]Broadly speaking, if we express a place proportionately ('half way down' the fleet, or 'three quarters down' the fleet) then the 'proportional' score is about the same for any fleet size. Given that the score for a last place is always 10.5, that the increment between places is $(\mathcal{S}_{2,n} - 10.5)/(n - 2)$ and that $\mathcal{S}_{2,n}$ rises quickly from 43 to just under 90 by $n = 14$, the 'proportional' score itself will rise more gradually as fleet size increases.

[10]If it is an issue, a simple expedient would be to generate more values by using the basic features of a minimum value of 10.5, a maximum of (about) 100.5, $\delta'_n = 1.25$, and Equations 1 and 2.

It seems odd that the maximum score is just over 100, although note that in practice this would only occur in a fleet size of 19 and above, which as suggested above, is not common.

If we 'adjust' the maximum score so that δ'_n is 1 rather than 1.25, we obtain scores which appear to come close to, but do not exceed 100; see Figure 4. It is plausible, but not certain, that the maximum score arises as an artifact of the 1.25 multiplier for the first position. See Figure 7 and the associated discussion.

2.1.3 Minimum scores

The minimum score is 10.5 (an apparently arbitrary figure). In a single boat race, first and last should 'converge'. After all, the boat is both first and last in that situation. However, this is not reflected in the Rinderle B result. But is a one boat race a race in any sense? While it would be reassuring a result at this limit was consistent, it is not too surprising that it is not, and it is a situation which is likely to occur infrequently.

2.2 The 'modified Rinderle B'

2.2.1 The algebraic solution

Downing (2005) introduces Equation 4 and states that 'Geoff Burrell . . . has supplied this algebraic representation of Rinderle B'. In fact Burrell (2007b) notes that he 'create(d) a mathematical fit to the tabulated values, insofar as this was possible, and came up with the values which are now used by the Denver Sailing Association.'

It is important to realise that this is a 'best fit' to the lookup table. Equally it is important to appreciate that Burrell makes two important changes:

- scaling the scores so that the maximum possible score is 100 (intuitively this seems much less arbitrary), which is a direct consequence of
- standardising the ratio of *all* differences between adjacent scores for a given fleet size to be unity (i. e. implicitly setting $\delta'_n = 1$).

The formula gives the points scored, $\mathcal{S}_{p,n}$, for *finishing* in position p in a race for which there are n starters as:

$$\mathcal{S}_{p,n} = (100 - d) \times \underbrace{\frac{n-p}{n-1}}_{t_1} \times \left(\underbrace{1 - e^{-(b+cn)}}_{t_2} \right) + d \tag{4}$$

For ease of reference, the terms are grouped and labeled[11] as ${}_0t_1$ and ${}_0t_2$. The

[11] We haved added the zero prefix subscript to indicate that this is the 'zeroth' version of Chips. As we introduce successive iterations this 'pre-script' will change to reflect the version. Note too that the ordering of the terms is reversed from Clark (2006).

Table 2: Parameter values for Denver Sailing Association and New England Multihull Association versions of modified Rinderle B; Chips1, Chips2 & Chips3. The ϵ' is a measure of the goodness of fit of the parameters to the Rinderle B lookup table. See Section 2.2.6 for details.

parameter	modified Rinderle B DSA	modified Rinderle B NEMA	modified Rinderle B here	Chips1	Chips2	Chips3
b	0.8	0.852	0.676	0.81	1.713	1.761
c	0.23	0.185	0.190	0.23	0.163	0.1622
d	10.5	10.5	10.5	10.5	10.0	5.0
f	–	–	–	1.478	–	–
k	–	–	–	1	–	–
Δ	–	–	–	–	–	15.3
ϵ'	262.9	55.75	4.93			

scores are intended to scale between d and 100. The range of scores and their maxima and minima are clearly dependent on the number of competitors. To put some values on the rankings, Figure 5 shows the scores produced from the DSA modified Rinderle B over a variety of fleet sizes, using the parameters from Table 2.

2.2.2 Non-finishers (and others)

The original Rinderle B lookup table does not explicitly define scores for positions like DNF. RYA (2004) scores DNF as $n+1$. Equation 4 permits us to find scores for $\mathcal{S}_{n+1,n}$. Unfortunately, if the fleet size is below 9, a DNF would score negatively, which seems rather punitive. This is obviously not a sensible strategy and we are left assigning a 'suitable' value for this situation.

A category like DNC is ideally scored as zero: in other words, not turning up gains no points at all, which meets the spirit of the requirement 'to provide an incentive to race'.

2.2.3 Simplification

One of the features of this type of scoring is that for a given fleet size it can be transformed into a simple linear equation. Each successive score is a constant increment (or decrement, or 'spread' in Bemis' terms) from the previous. As described by Downing (2005) it is 'linear with respect to place'. Some algebraic manipulation gives the difference between places, δ_n, for a

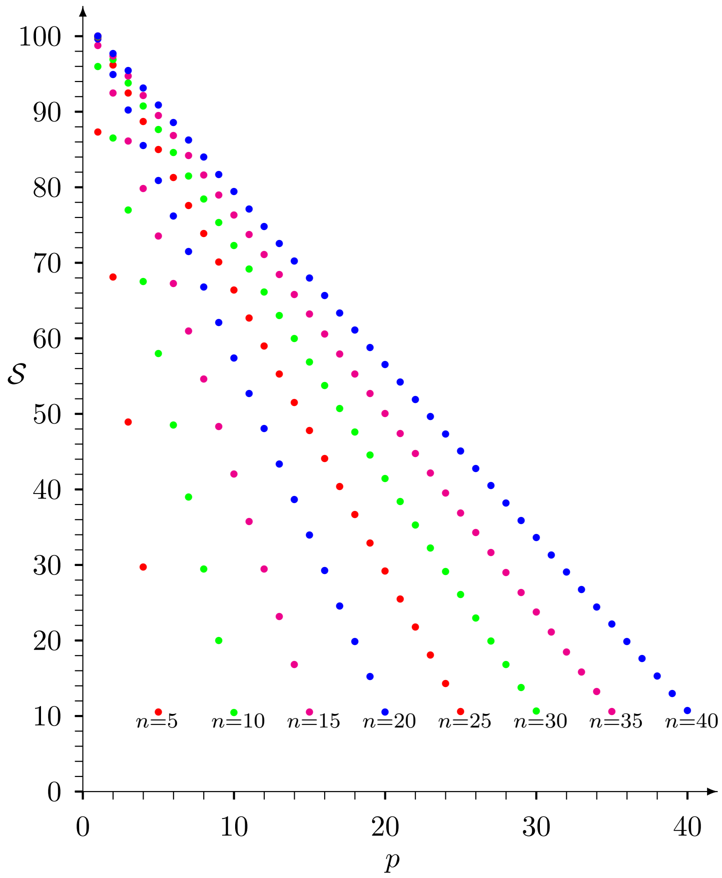

Figure 5: Scores for place p in selected fleet sizes, calculated from modified Rinderle B (Equation 4, DSA parameters)

given fleet size n as

$$\mathcal{S}_{p,n} - \mathcal{S}_{p+1,n} = \delta_n = \frac{(100 - d)}{n - 1} \times \left(1 - e^{-(b+cn)}\right) \tag{5}$$

Although the origin can be conveniently evaluated with respect to the first place, it is easier to evaluate with respect to the *last* place, since this has a fixed value of d, saving us any further calculation. Therefore

$$\mathcal{S}_{p,n} = d + \delta_n(n - p) \tag{6}$$

2.2.4 An aside

Note that a fleet size of 1 gives an indeterminate value for the score since we have a division by $n - 1$ (i. e. zero).

A way out of the indeterminacy would be to change the expression very slightly so that the ${}_0t_1$ term has a meaningful value. One suggestion is to add 1 to the numerator and denominator, so that the term becomes $\frac{n-p+1}{n}$. This requires we rewrite Equation 4 as:

$$\mathcal{S}_{p,n} = (100 - d) \times \frac{n - p + 1}{n} \times \left(1 - e^{-(b+cn)}\right) + d \tag{7}$$

Clearly this will alter the value of $\mathcal{S}_{n,n}$. This also resolves the issue with DNF, since $\mathcal{S}_{n+1,n}$ would then be d. The differences δ_n change to become

$$\delta_n = \frac{(100 - d)}{n} \times \left(1 - e^{-(b+cn)}\right) \tag{8}$$

The impact of these changes would be felt most with small fleet sizes.

2.2.5 A digression: an alternative expression

An alternative version of modified Rinderle B to equation 4 is given in NEMA (2000?):

$$\mathcal{S}_{p,n} = 100 \times \left(\frac{n - p}{n - 1} \times \left(1 - e^{-\frac{n+4.6}{5.4}}\right)\right) + d \tag{9}$$

Again this appears to be an algebraic fit to the table of values. We might (rightly) be suspicious of the 100 multiplier, expecting it to be $(100 - d)$. As expected, this formulation leads to values which exceed 100. Looking at results published by the New England Multihull Association, who may have used modified Rinderle B in (at least) 2000 and 2001, some scores do exceed 100, but at most they rise as high as 100.3. It is therefore not clear whether they were using the lookup table or the modified equation. Restoring the $(100 - d)$ multiplier and rewriting the $e^{-\frac{n+4.6}{5.4}}$ term to a more 'familiar' $e^{-(b+cn)}$ form where $b = 0.852$ and $c = 0.185$ yields quite reasonable results.

2.2.6 Fit of the parameters

How well do the equations fit the scores in the lookup table? Through Equation 4 and the DSA and NEMA parameters in Table 2 we can generate tables and compare them to Table 1. Rather than wade through reams of numbers, we can write

$$\epsilon_{p,n} = \mathcal{S}_{p,n} - \mathcal{S}^*_{p,n} \tag{10}$$

where $\mathcal{S}^*$ is the score using an equation, and calculate

$$\epsilon' = \sum_{i=2,n} \sum_{j=1,i} \epsilon^2_{i,j} \tag{11}$$

(the sum of the squared deviations) to give some measure of fit. The $\mathcal{S}_{1,n}$ scores have been deliberately omitted in this comparison, since neither equation takes the 1.25 multiplier into account. Of course this is a rather crude measure, since it does not highlight systematic deviations.

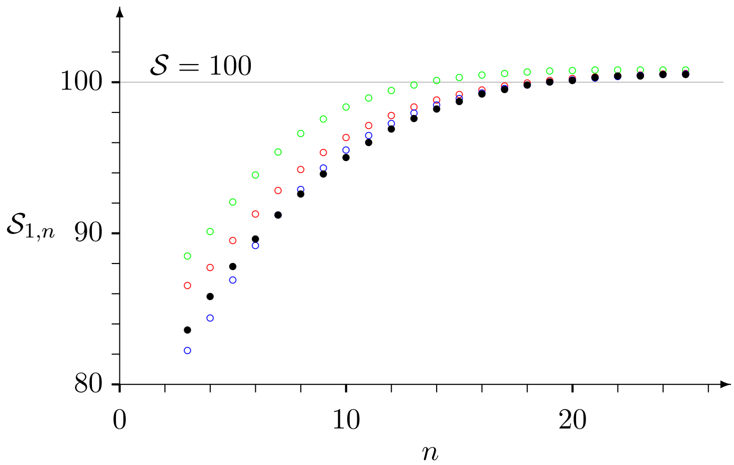

Figure 6: Values for first place: • from Rinderle B lookup table; ∘ from DSA parameters; ∘ from NEMA parameters; ∘ from parameters found earlier (page 13).

The DSA parameters give a value of ϵ' of 262.9, while the NEMA parameters give 55.75. However, a little further experimentation gives $\epsilon' = 4.93$ when $b = 0.68$ and $c = 0.19$.

Inspection of the results for the best fit reveal that the most significant deviations occur where the value of n is low. Once we have fleet sizes over 5, the differences are almost all less that 0.1. Given that all the observed values are presented to the nearest 0.1, this seems reasonable. The largest deviation, of 0.68, is found for $\mathcal{S}_{2,4}$.

As a further check on the fit of these parameters to the lookup table we can compare the given and calculated values for $\mathcal{S}_{1,n}$, including the 1.25 multiplier on δ_n:

$$\mathcal{S}_{1,n} = \mathcal{S}_{2,n} - 1.25\delta_n \tag{12}$$

Figure 6 shows the scores for first for the three sets of b and c values, and the scores found from the lookup table. Note that the values for fleet sizes of 1 and 2 are not presented. As we have discussed earlier, Equation 4 is indeterminate for a fleet size of 1, and when the fleet size is 2, it is unclear how to deal with the multiplier.

However, this raises a small problem. If we retain this 1.25 'incentive' for first we find that as the fleet size increases, the score for first increases to a maximum, and then slowly declines. Figure 7, which uses the parameters derived here shows the general trend. A maximum of 100.62 is found when the fleet size is 31. As the fleet size increases the maximum score declines gradually towards 100.0. Using the DSA parameters, the maximum is 100.8 at $n = 23$, while the NEMA parameters give a maximum of 100.62 at $n = 30$. The lookup table itself has a maximum value of 100.5 at $n = 25$.

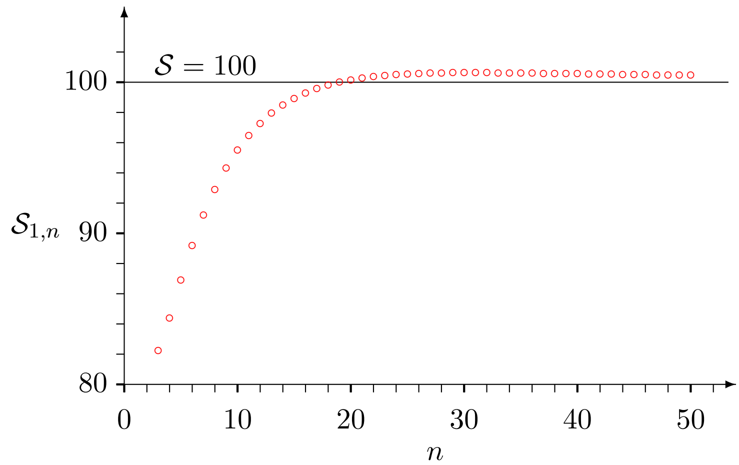

Figure 7: Values for first place for larger fleet size, from parameters found earlier (page 13). The maximum score of 100.62 is found when $n = 31$. The other parameters give similar results.

In real life this is unlikely to be of significance. The fleet size would have to be fairly large and the difference in scores between a first from a fleet size of 31 and one of 60 is only 0.2 (to one decimal place). Still, it would be difficult to explain to someone that their first place is worth less as the fleet size increases (beyond a certain size). Such behaviour, though mathematically sound, is counter-intuitive and does not instill confidence.

2.2.7 Sensitivity

Just how sensitive are the results to changes in b and c? The basic form of the equation indicates that ${}_0t_1 \times_0 t_2$ is designed to scale between 0 and 1, so that the scores fall between d and 100. We need only examine the behaviour of ${}_0t_1$ and ${}_0t_2$. The first term, $\frac{n-p}{n-1}$ itself scales in the interval $[0, 1]$, which only leaves the behaviour of ${}_0t_2$ to be examined. Again, its form indicates that $e^{-(b+cn)}$ is intended to take values between 0 and 1. With that constraint, as n increases, ${}_0t_2$ approaches zero. Once $n > c^{-1} + b$ it starts to swamp the effects of c and b. The smaller the value of c and the larger the value of b, the less the effect of n.

Figures 8 and 9 attempt to assist in visualising the effects of varying b and c. The values held constant are close to those in Table 2 but the other values chosen are fairly extreme in order to make the effects obvious. In Figure 8, holding the value of b constant at 0.7, low values of c give a 'straighter' line. In terms of scores, this translates to a more gradual change in the value of δ_n with increasing fleet sizes.

Taking a 'reasonable' value of $c = 0.2$ in Figure 9, it is apparent that while the asymptote is approached at about the same value of n by both val-

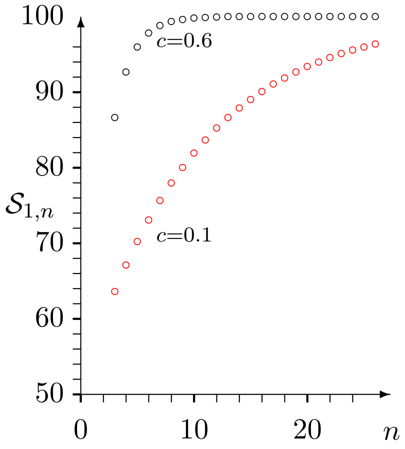

Figure 8: First place scores for $b = 0.7$, varying c. As c increases in value, the curve rises to its asymptote more quickly.

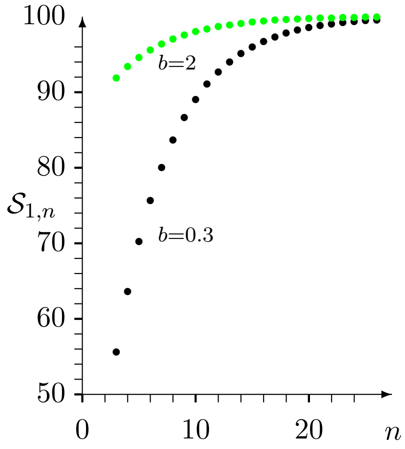

Figure 9: First place scores for $c = 0.2$, varying b. The higher the value of b the tighter the spread of the scores.

ues of b, a low value of b makes the initial slope much steeper, and therefore δ_n is more sensitive to fleet size.

These figures are really modelling the behaviour of $1 - e^{-(b+cn)}$. The ${}_0t_1$ term is 1, and the other parts of Equation 4 just change the scaling and origin. With that in mind, it is fairly obvious that as n increases it will swamp the influence of both b and c. The smaller the value of c, the higher the value of n required before this effect will dominate.

2.3 Rinderle B revisited

Perhaps we are being altogether too clever, or perhaps just too 'algebraic' too see the wood from the trees. Ignoring the 1.25 multiplier for the moment, re-assign the $\mathcal{S}_{1,n}$ values to be $\mathcal{S}_{2,n} + \delta_n$. Rinderle B distributes the scores linearly between first and last. Last place has a static score of 10.5. If we have a convenient function which gives a score for first, then it is a simple matter to work out the increment and therefore any given score. What characteristics do we want from the 'convenient function'? The single most important characteristic is that it is monotonically increasing, and preferably asymptotic. While we want scores to increase as the fleet size increases, we wish them to rise to some known limit.

Although many functions will do, a simple, parsimonious one is the most attractive. Scaling the scores between d and 100 as we have done up to now, a simple exponential like

$$\mathcal{S}_{1,n+1} = d + (100 - d) \left(\frac{\mathcal{S}_{1,n}}{100} \right)^{\alpha} \tag{13}$$

can provide a suitable model. The starting point, $\mathcal{S}_{1,1}$ can be chosen to make some sort of sense: a value which is somehow reasonable for a one boat race. The α controls the curvature as the function rises towards its asymptote. If a fleet size of (say) 25 is anticipated, then effectively reaching the asymptote at around 25 would be desirable.

A starting point of 66.2 and a value of α of 0.927 gives a fair fit to the existing Rinderle B data,[12] but frankly fitting the data is not too important, provided the function fulfills the criteria. The 66.2 seems reasonable as about two thirds of the maximum score, while the exponent ensures we are only 0.5 from the maximum score with a fleet of 25. But other starting points and other exponents could be equally suitable, depending on finding some suitable justification.

A slightly simpler model also provides a good fit:

$$\mathcal{S}_{1,n+1} = 100 \left(\frac{\mathcal{S}_{1,n}}{100} \right)^{\alpha} \tag{14}$$

This time, a starting point of 66.8 and an α of 0.827 is suitable. The other obvious model is

$$\mathcal{S}_{1,n} = (100 - d) \left(1 - e^{-(b+cn)} \right) + d \tag{15}$$

which also fits at least as well, with $b = 0.79$ and $c = 0.175$. Here the score for $\mathcal{S}_{1,1}$ is 65.7. In passing, we might have expected the parameters to be closer to one of those in Table 2, but here we have been fitting the curve only to the $\mathcal{S}_{1,n}$ values, while those in the table were fitted to all the available data. In addition we are using $d = 10$ rather than 10.5.

All these models are satisfactory, and are so close that plotting all three reveal merely shows how very similar they are (they are very similar to the 'adjusted' firsts of Figure 4. They each give a very similar starting point and come reasonably close to the maximum value by $n = 25$. This is the sort of behaviour we want. But, in the absence of any theoretical justification, a variety of other equations or parameters would be as appropriate.

The δ_n are easily found, as $(\mathcal{S}_{1,n} - d)/(n-1)$, and therefore any place in a given fleet size is found as $\mathcal{S}_{1,n} - (p-1)\delta_n$. A couple of simple, relatively easily explained equations give the scores from either one or two parameters.

2.3.1 Let Rinderle B embrace DNF

Let us adopt the fairly standard notion that a DNF scores as $n + 1$. As noted earlier simply applying this blindly as $\mathcal{S}_{n,n} - \delta_n$ is not desirable. It is especially unsatisfactory with small fleets where negative scores would

[12] In fitting these functions, a value of $d = 10$ rather than 10.5 was used. The 10.5 value seems unduly arbitrary. But it does underscore the fact that it is the form of the curve rather than its (or their) exact realisation which is critical.

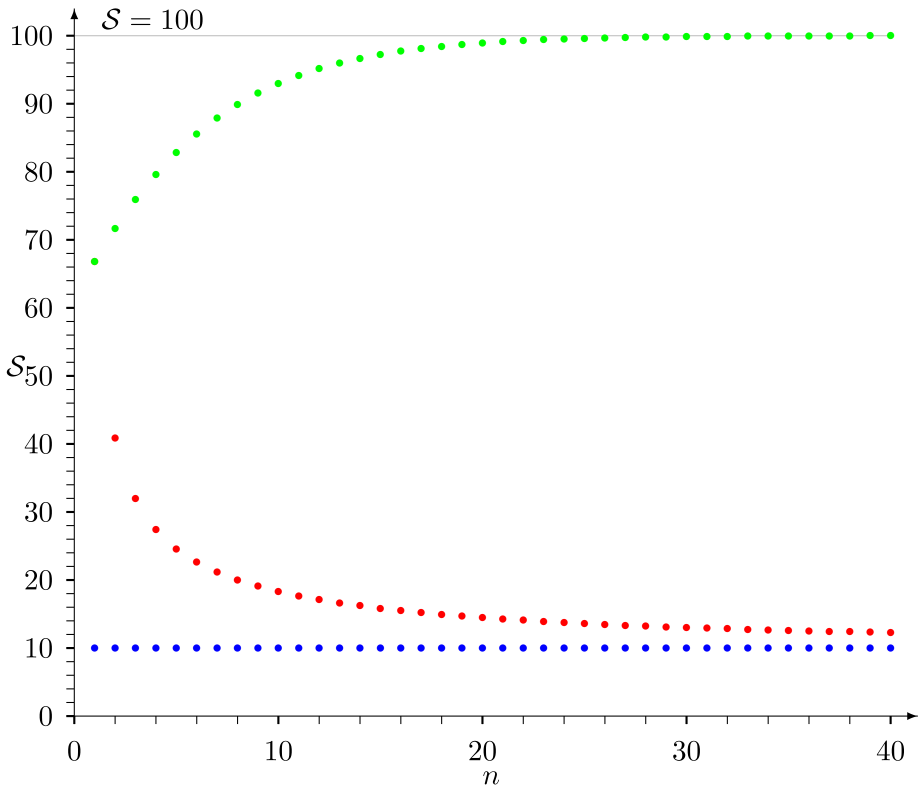

Figure 10: First •, last • and DNF • places scored by Equation 14. Note that in a one boat race, first and last scores the same (by definition).

arise – no incentive at all to race. Adopting the model of Equation 14, we calculate δ_n as $(\mathcal{S}_{1,n} - d)/n$, in other words, scaling the increment over an 'extra' place we will find that the DNF score is what our former 'last place' score was (i. e. d). This has a rather pleasant side effect: the last place scores are no longer a constant, but form a curve similar to that shown in Figure 10. This helps to mitigate a rather harsh feature of Rinderle B, and seems (to me) to provide a greater incentive to race.

Obviously, d does not have to be the value we have used up to now. If we felt that a DNF was worth a score of 5, we could as easily adjust to accommodate this.

Another welcome side effect is that in a one boat race there is a score for failing to finish, something absent in the regular Rinderle B formulation.

3 Chips

3.1 Version 1

Geoff Burrell took the modified Rinderle B 'exponential' formulation to create the first version of his scoring scheme for Chipstead Sailing Club (Burrell, 2004?). Burrell (2007b) states that one of his objectives was to adjust the scores of the 'bottom' of the fleet:

> I believe Rinderle B is far too aggressive in the way it penalises boats further down the fleet when there are only a few competitors.

Rinderle B makes no distinction between coming last in a 5 or 40 boat race. This is true of the lookup table as well as the modified form. Implicitly, this CHIPS formulation rewards getting out there on the water and sailing. Intuitively this is attractive.

The formulation of this first version was:

$$\mathcal{S}_{p,n} = (100 - d) \times \left(\underbrace{\left[\frac{n-p}{n-1}\right]^k}_{{}_1t_1} \times \left(\underbrace{1 - (1+f)e^{-(b+cn)}}_{{}_1t_2} \right) + \underbrace{fe^{-(b+cn)}}_{{}_1t_3} \right) + d \tag{16}$$

This looks initially formidable, but examination shows it is generically close to his formulation of Rinderle B. The chief differences are

- the introduction of an exponent k on the ${}_1t_1$ term, $\frac{n-p}{n-1}$
- the $(1 + f)$ scaling factor within ${}_1t_2$
- and the additional expression $fe^{-(b+cn)}$

The exponent k has the effect of removing the linearity of δ_n. When it is below 1 the intervals between adjacent scores will be greatest at the 'bottom' of the fleet, and when it is above 1, the intervals increase towards the top of the fleet, providing a greater 'incentive' to higher positions, especially first. The introduction of k would generate a 'variable and increasing' scoring system, in Bemis's (1960) terms. To demonstrate the effect of the exponent simply it is introduced into Equation 4, as ${}_0t_1^k$

$$\mathcal{S}_{p,n} = (100 - d) \times_0 t_1^k \times_0 t_2 + d \tag{17}$$

the scores for a fleet size of 20 are given in Figure 11.

However, the exponent is not really an issue, since its suggested value in Table 2 is 1. Burrell (2007a) notes that the purpose of the parameter was "to create an increased score advantage to those at the top of the fleet" but that it had "since (been) discovered to have no great merit".

When $f = 0$, the expression obviously reverts to its simpler expression. Changes to f have the effect of altering the value of $\mathcal{S}_{n,n}$, and hence δ_n.

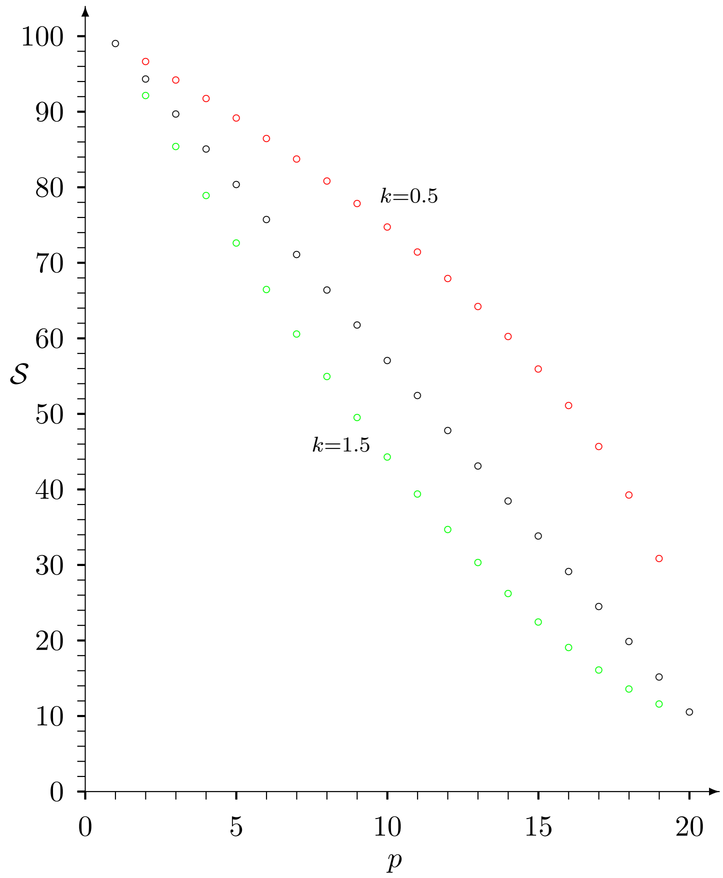

Figure 11: Scores for place p in fleet sizes of 20, calculated from a modified 'modified Rinderle B' (Equation 17) to demonstrate the effect of an exponential scaling. The scores for first and last will be identical, whatever exponent is chosen.

The introduction of this parameter was the principal way in which CHIPS removed the penalty for low positions in small fleets. Instead of a miserly 10.5, last place received a higher reward. It is the introduction of the parameter f which provides CHIPS with its major step forward. A cursory glance at Equation 16 indicates that f has no effect on the value for first place.

Figure 12 shows the scores for first and last for fleet sizes up to 20 when f is varied from 0.5 through to 2 in 0.5 increments. As discussed above, increasing f increases the scores for last. More importantly, it 'scales' the increase with fleet size. There is an exact relationship between f and the scores for first and last:

$$f = \frac{100 - \mathcal{S}_{(1,n)}}{(\mathcal{S}_{n,n} - d)} \tag{18}$$

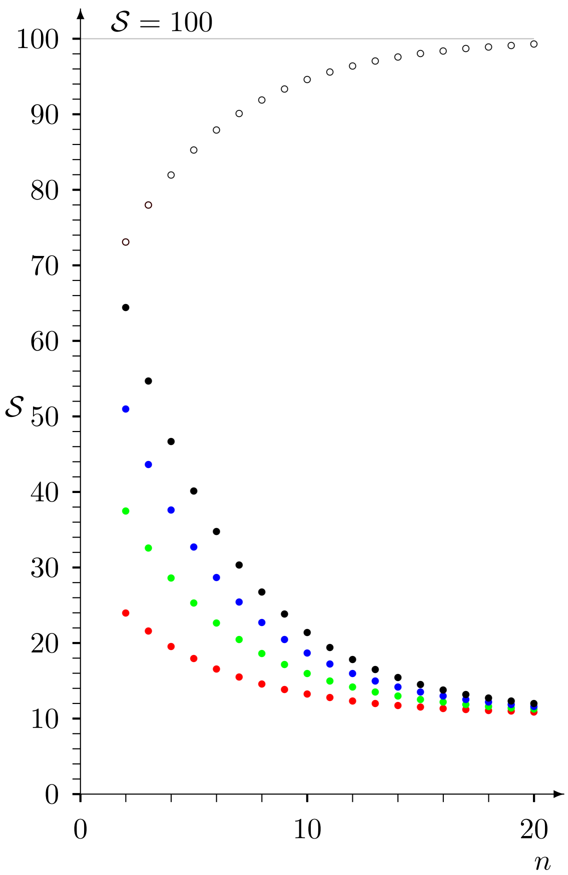

Figure 12: Changing the value of f. The open circles show the score for first place. The closed circles are the scores for last place for $f = 0.5 (lowest curve), 1.0, 1.5, 2.0$.

Comparing this figure with Figure 10 reinforces the similarity of the curves when $f = 1$.

It may be worth pointing out that the same ideas of an 'origin' and 'scale' (*spread* or δ_n) which allow us to calculate values of $\mathcal{S}$ for a given fleet size in Equation 5 are also present here. The scale is simply modified to take account of the f term:

$$\delta_n = (100 - d) \times \frac{1 - (1 + f)e^{-(b+cn)}}{n - 1} \tag{19}$$

This provides another way of looking at the influence of f. If we calculate a hypothetical 'origin' as $\mathcal{S}_{0,n}$ (a 'zeroth' place, which obviously has no physical meaning, but makes calculation more intuitive) from the full equation, then any $\mathcal{S}_{p,n}$ is simply

$$\mathcal{S}_{p,n} = \mathcal{S}_{0,n} - p\delta_n \tag{20}$$

In this first 'iteration' which was trialed at Chipstead Sailing Club in 2005, the scores for the places in a three boat race were conveniently rounded to 40, 60 and 80.

3.2 A second iteration

This initial Chips formulation was replaced by Chips2, which simplified the equation by dropping the exponential term k, and eliminating f. Burrell's intention in removing f was prompted by his wish to remove the "illogical double score for a single-boat race".[13] In reality this is seldom an issue but it is an inelegant mathematical anomaly. He did this by making the first and last place score equal when $n = 1$:

$$f = e^{b+c} - 1 \tag{21}$$

When $n = 1$, the ${}_1t_1$ term is $\frac{0}{0}$ which is rather uncomfortable mathematically, but to see what is really going on, let us concentrate on the ${}_1t_2$ and ${}_1t_3$ terms. But first, note that an easy way out of the 'division by zero' indeterminacy is to redefine n by letting it be the number of starters *plus* 1. This is making explicit what we did in Section 2.3.1, and helps provide additional backing for the notion.

There are at least two ways of evaluating $\mathcal{S}_{1,1}$. In the first we take the plausible solution to the t_1 term that if $p = 1$ it is simply $\frac{n-1}{n-1}$. Remembering that $n = 1$ and ignoring the scaling and location terms involving d, we are simply now looking at

$$1 - (1 + f)e^{-(b+c)} + fe^{-(b+c)} = 1 - e^{-(b+c)} \tag{22}$$

If, on the other hand we decide that the t_1 term evaluates to zero, then we are looking at a simpler expression, and if we adopt Burrell's suggestion of equation 21, then we have

$$\begin{aligned} fe^{-(b+c)} &= (e^{b+c} - 1)e^{-(b+c)} \\ &= 1 - e^{-(b+c)} \end{aligned}$$

which provides a nice symmetry (or perhaps circularity).

The formulation of Chips2 is therefore exactly the same as that of Chips1 but with the substitution of Equation 21 for f,

$$\begin{aligned} \mathcal{S}_{p,n} &= (100 - d) \\ &\times \left(\frac{n-p}{n-1} \times \left(1 - (1 + e^{b+c} - 1)e^{-(b+cn)} \right) + (e^{b+c} - 1)e^{-(b+cn)} \right) + d \end{aligned}$$

[13] personal communication, 17/01/2007

which, with some simplification of terms gives

$$\mathcal{S}_{p,n} = (100-d) \times \left(\underbrace{\frac{n-p}{n-1}}_{_2t_1} \times \underbrace{(1-e^{-c(n-1)})}_{_2t_2} + \underbrace{e^{-cn}}_{_2t_3} \times \underbrace{(e^{c}-e^{-b})}_{_2t_4} \right) + d \quad (23)$$

where we have labelled the terms as ${}_2t_{1\ldots4}$ to distinguish them from the terms used in the CHIPS1 equation. The ${}_2t_4$ term is a constant, although it is useful at this stage to retain it in full to show its derivation. Equations 16 and 23 are identical, taking into account the elimination of the constant f.

Besides making these alterations to the form of the equation, Burrell took the opportunity to change the constants b and c to 'flatten' the curves, reducing the dependence on the number of boats. He also took the opportunity to change the constant d. The values adopted are given in Table 2.

One result of the reformulation was that the scores in a three boat race now become 65, 77.5 and 90.

Substituting the values in the table into Equation 23 provides a more compact (if less elegant) form:

$$\mathcal{S}_{p,n} = 90 \left(\frac{n-p}{n-1} \left(1 - 1.177e^{-0.163n}\right) + 0.99671e^{-0.163n} \right) + 10 \quad (24)$$

CHIPS2 was the subject of Clark (2006). In it, a minor mathematical anomaly was pointed out, but the main thrust of the analysis was to point out that computationally it was advantageous to reformulate the equation into a form which took advantage of the fact that for a given fleet size, the intervals ('spread') between scores was constant (using the same approach which led to Equations 5 and 20 earlier).

The results generated by the different formulations of CHIPS 1 and 2 are shown for small fleet sizes in Figure 13. The values are starting to converge by the time the fleet size rises to 20, but for small values they are quite different.

3.3 Further refinement

Burrell (2007c) has more recently developed the form of the equation, in part to remove the anomalies reported earlier, but also to introduce other minor refinements. Although Burrell recasts the equation by recombining the ${}_2t_3$ and ${}_2t_4$ elements slightly, the basic form of the equation remains unchanged. The 'original' form was:

$$e^{-cn} \times (e^{c} - e^{-b}) \quad (25)$$

which is algebraically identical to

$$e^{-c(n-1)} - e^{-b-cn} \quad (26)$$

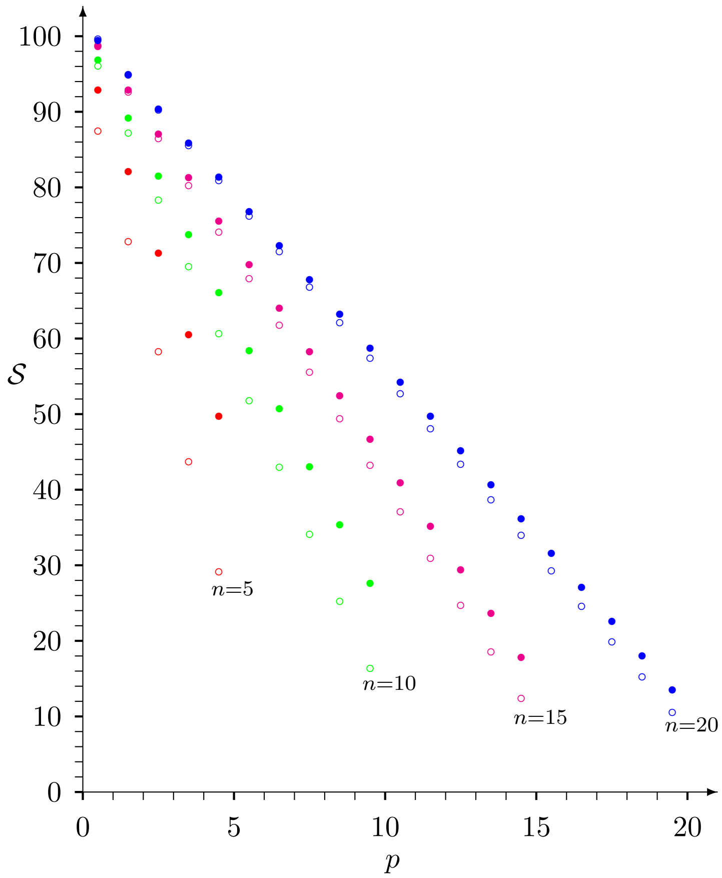

Figure 13: Scores for place p in selected fleet sizes, calculated from CHIPS1 (open circles) and CHIPS2 (closed circles).

Burrell changes the ${}_2t_1$ term as suggested in equation 7. It was this proposal by Burrell which led to the change in equation 7. This has the pleasing by-product of removing the 'divide by zero' issue when $n = 1$. But the main purpose here was to make the RTD score consistent by ensure that it behaved in a monotonic fashion.

The second change was to introduce a new term

$$\Delta = \mathcal{S}_{1,1} - \mathcal{S}_{2,1} \tag{27}$$

which is equivalent to δ_1. This enables the RTD score for a single boat race to be set. The CHIPS3 equation becomes:

$$\mathcal{S}_{p,n} = (100 - d) \times \left(\underbrace{\frac{n-p+1}{n}}_{{}_3t_1} \times \underbrace{\left(1 - \kappa e^{-c(n-1)}\right)}_{{}_3t_2} + \underbrace{\kappa e^{-c(n-1)}}_{{}_3t_3} - \underbrace{e^{-b-cn}}_{{}_3t_4} \right) + d \tag{28}$$

Again we have labelled the terms as ${}_3t_{1\ldots4}$ to distinguish them from the terms used in the other CHIPS equations. If we use equation 28 to calculate $\mathcal{S}_{1,1} - \mathcal{S}_{2,1}$ we note that many of the terms evaluate to either 0 or 1 and we find that

$$\delta_1 \text{ (or } \Delta) = (100 - d)(1 - \kappa) \tag{29}$$

In passing, the general term for δ_n is

$$\delta_n = \frac{(100 - d)(1 - \kappa e^{-c(n-1)})}{n} \tag{30}$$

and Equation 29 is just the special case when $n = 1$.

It does not matter whether the term κ appears in Equation 28 as part of ${}_3t_3$ or not: δ_1 (and Equation 30) take the same values. Do we need it included in ${}_3t_3$? It is not immediately clear why it is there. Burrell does not offer an explanation. Evaluating the scores for small fleet sizes (i. e. up to 10) with and without the κ term on ${}_3t_3$ shows that it is needed in order to ensure that the maximum scores, $\mathcal{S}_{1,n}$, lie within reasonable bounds. Without it, the values are just too high. Its contribution reduces with the value of n. The particular values of the constants chosen by Burrell ensure that there is only a minimal difference with the previous tables to the extent that when added to Figure 13 they overlie the CHIPS2 values. By the time $n = 20$ some divergence is noticeable, but it is very small. The envelope of scores is shown in Figure 14. It may be instructive to compare this with Figure 10.

3.3.1 Different strokes

Burrell presents the CHIPS3 equation as

$$\begin{aligned}\mathcal{S}_{p,n} &= (100 - d) \\ &\times \left[\frac{n + 1 - p}{n} \times \left(1 - 0.9866682e^{-0.1622n}\right) + 0.81475e^{-0.1622n}\right] + d\end{aligned}$$

which slightly begs the question of where the parameters b, c and δ (or κ) have gone. This is a computational form, where a great deal of simplification has gone on. In some ways it obscures the evolution of the equation. However it is equivalent and produces the same scores. It is in this sort of form to make it easy to include in Sailwave (Jenkins, 2006) as a 'Custom high point defined by a formula'. Since then, CHIPS3 has been fully integrated within Sailwave, and the simplification is not as necessary. Nevertheless, if it is necessary to calculate scores, it provides a useful starting point.

Exploiting the 'variable spread' nature of the expression, for any fleet size it necessary only to calculate the 'starting point' and the δ_n, *cf.* Equation 6. However for preference I would use a starting point of $\mathcal{S}_{0,n}$. Finding a score for coming in *before* the first boat may seem rather counter intuitive, but it

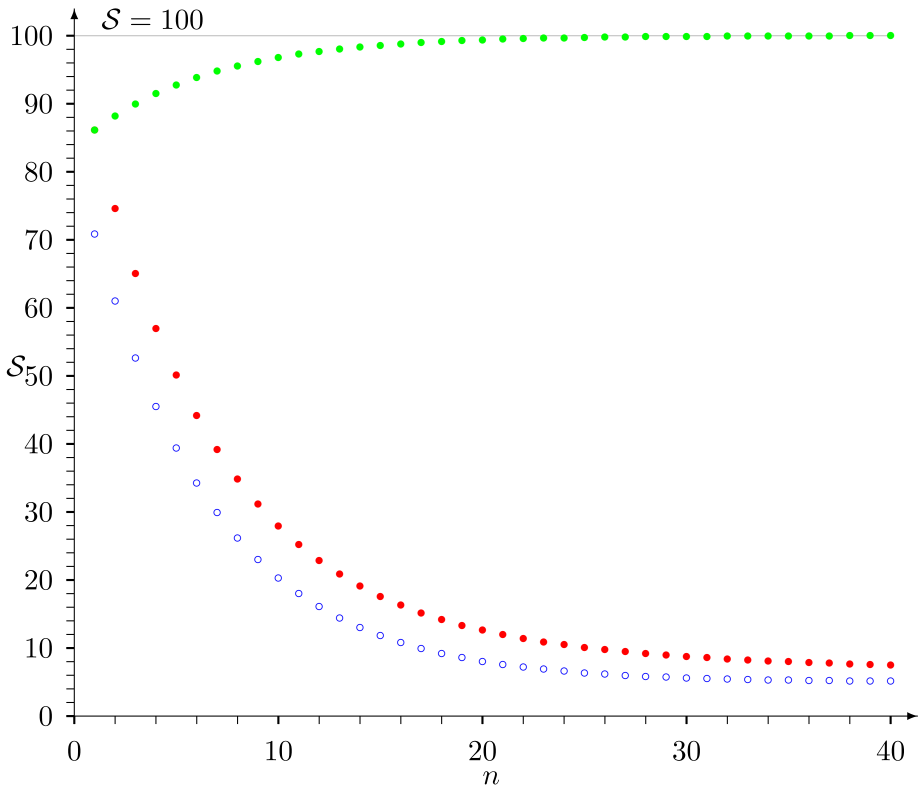

Figure 14: First •, last • and DNF ∘ places scored by CHIPS3, Equation 28. Note that in a one boat race, first and last scores the same (by definition). Compare with Figure 10.

is mathematically sound, and the advantage it confers is that any score is now found by subtracting 'place' times 'spread' from the starting point:

$$\mathcal{S}_{p,n} = \mathcal{S}_{0,n} - p\delta_n \tag{31}$$

which *may* be simple enough to explain to the great horde who claim 'I was never any good at maths'. If finding $\mathcal{S}_{0,n}$ and δ_n represents a challenge, Table 3 may help. Of course, Burrell's table makes things even easier (for fleet sizes up to 25), provided you can follow columns and rows! Table 3 may require a word of reassurance. The $\mathcal{S}_{0,n}$ terms do exceed 100 and they are not monotonic, rising to a maximum at $n = 12$ and then falling away to an asymptote of 100 at $n = \infty$. This is simply an artefact and need not cause any concern. The critical value

$$\mathcal{S}_{1,n} = \mathcal{S}_{0,n} - \delta_n \tag{32}$$

never exceeds 100. Its theoretical maximum, at $n = \infty$, is 100. In terms of

Table 3: Values of $\mathcal{S}_{0,n}$, the 'starting value', and δ_n, the 'spread' for fleet sizes up to 50.

n	$\mathcal{S}_{0,n}$	δ_n	n	$\mathcal{S}_{0,n}$	δ_n	n	$\mathcal{S}_{0,n}$	δ_n	n	$\mathcal{S}_{0,n}$	δ_n	n	$\mathcal{S}_{0,n}$	δ_n
1	101.42	15.30	11	104.46	7.21	21	103.83	4.38	31	102.94	3.04	41	102.29	2.31
2	101.81	13.62	12	104.47	6.80	22	103.74	4.20	32	102.86	2.95	42	102.24	2.26
3	102.42	12.46	13	104.45	6.43	23	103.64	4.03	33	102.79	2.87	43	102.19	2.21
4	102.97	11.50	14	104.41	6.09	24	103.55	3.88	34	102.72	2.78	44	102.14	2.16
5	103.41	10.67	15	104.35	5.78	25	103.45	3.74	35	102.65	2.71	45	102.10	2.11
6	103.76	9.93	16	104.28	5.50	26	103.36	3.60	36	102.58	2.63	46	102.05	2.06
7	104.02	9.27	17	104.20	5.24	27	103.27	3.48	37	102.52	2.56	47	102.01	2.02
8	104.21	8.67	18	104.12	5.00	28	103.18	3.36	38	102.46	2.49	48	101.97	1.98
9	104.34	8.14	19	104.02	4.77	29	103.10	3.25	39	102.40	2.43	49	101.93	1.94
10	104.42	7.65	20	103.93	4.57	30	103.02	3.14	40	102.35	2.37	50	101.89	1.90

values rounded to a single place of decimals a score of 100 is encountered at $n = 57$.

4 Conclusion

Many already claim that scoring schemes like these are 'too complex', although we don't reject flying or a prestige office block because the mathematics of aerodynamics or stress analysis is 'too complex'. By comparison, on a scale of 1 to 10, modified Rinderle or CHIPS doesn't even register. But if a fleet is prepared to accept a lookup table, it is straightforward to generate a suitable table based on one of the equations. No-one has to know the mathematical niceties which underly the table; they just have to be confident that the scoring does reward them appropriately (or perhaps, not penalise them adversely). Equally, the use of electronic gadgetry is common in sailing, and lookup tables may be rather passée. Burrell has published an

Excel spreadsheet which calculates the score for any combination of position and fleet size. The spreadsheet also helps the user to determine what results are needed to beat their fellow competitors. Although this is written in Excel, it would be straightforward to turn it into Java and make it independent of proprietory software and therefore make it capable of running on many platforms.

An advantage of CHIPS, which must not be overlooked, is the exceedingly small probability that tied scores will result. A short series might give two very close scores, and that would be mainly the result of CHIPS rounding to the nearest tenth of a point. Most of the schemes regularly used to resolve ties are unsatisfactory in one way or another. CHIPS sets us free.

References

Robert N. Bavier, Jr. *Faster Sailing.* Nicholas Kaye Ltd., 1956.

F. Gregg Bemis. *Yacht Race Scoring.* privately published, 1960. printed in Boston.

Geoff Burrell. All about "CHIPS" (Chipstead High Point Scoring). Chipstead Sailing Club website, February 2007a. URL www.chipsteadsc.org.ok.

Geoff Burrell. CHIPS implementation in Sailwave. personal communication, January 2007b.

Geoff Burrell. Definition of "CHIPS" (Chipstead High Point Scoring). Chipstead Sailing Club website, March 2006. URL www.chipsteadsc.org.uk.

Geoff Burrell. Definition of "CHIPS" (Chipstead High Point Scoring). Chipstead Sailing Club website, March 2007c. URL www.chipsteadsc.org.ok.

Geoff Burrell. All about "CHIPS" (Chipstead High Point Scoring). Chipstead Sailing Club website, 2004? no longer available.

Malcolm Clark. The Chipstead High Point Scoring system in practice. November 2006. URL www.styvechale.net/chip.pdf.

Tom Downing. High point scoring comes to DSA. *Denver Sailing Association Newsletter*, page 1 and 3 (of 8), March 2005. URL www.denversailing.org.

GMORA. Rinderle B Scoring System. website, undated. URL www.gmora.org/rinderle_b.asp. Gulf of Maine Ocean Racing Association.

Colin Jenkins. Sailwave, Sailing Scoring System, 2006. URL `www.sailwave.com`.

NEMA. Rinderle B Point Scoring. (New England Multihull Association) sent by Tom Downing, 2000?

RYA. *RYA Racing Rules of Sailing 2005–2008*. Royal Yachting Association, RYA House, Ensign Way, Hamble, Southampton, SO41 4YA, October 2004. ©ISAF June 2004.

`malcolm at styvechale.net`